

# IM 3

# 4-1 Even and Odd Functions

Name: \_\_\_\_\_ Per: \_\_\_\_\_ Date: \_\_\_\_\_

## Complete the following.

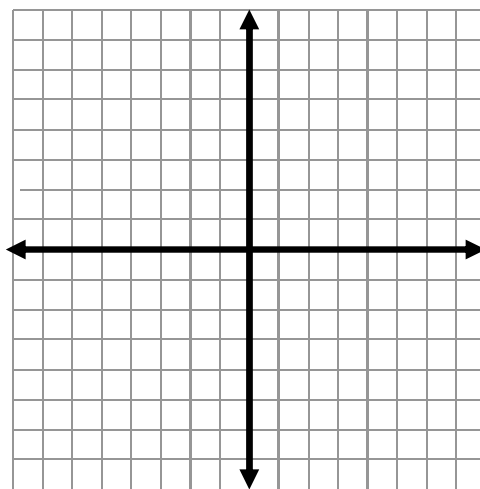
Some functions can be classified as *even* or *odd* functions. You can tell by looking at the graph of the function or by using algebra.

An **even function**, say  $f$ , is a function where  $f(-x) = f(x)$ . The graph is symmetric across the  $y$ -axis. So if the point  $(x, y)$  is on the graph of  $f$ , so is the point  $(-x, y)$ .

An **odd function**, say  $f$ , is a function where  $f(-x) = -f(x)$ . The graph is symmetric about the origin. So if the point  $(x, y)$  is on the graph of  $f$ , so is the point  $(-x, -y)$ .

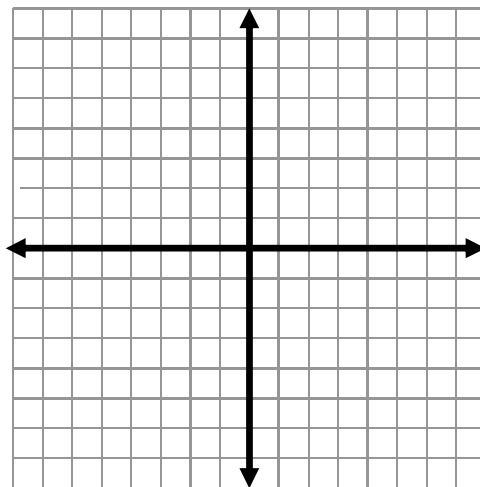
**Example 1.** Determine whether the function  $f(x) = x^4 - 4x^2$  is *even*, *odd*, or *neither* by graphing.

1. Type the function into Desmos.
2. Sketch the graph on the grid to the right. Make your graph accurate; include any  $x$ -intercepts (zeros) the function has.
3. Describe any symmetry. \_\_\_\_\_  
\_\_\_\_\_
4. What direction is the left end going? \_\_\_\_\_
5. What direction is the right end going? \_\_\_\_\_



**Example 2.** Determine whether the function  $f(x) = x^3 - 4x$  is *even*, *odd*, or *neither* by graphing.

1. Type the function into Desmos.
2. Sketch the graph on the grid to the right. Make your graph accurate; include any  $x$ -intercepts (zeros) the function has.
3. Describe any symmetry. \_\_\_\_\_  
\_\_\_\_\_
4. What direction is the left end going? \_\_\_\_\_
5. What direction is the right end going? \_\_\_\_\_



**Example 3.** Determine whether the function  $f(x) = x^4 - 4x^2$  is *even*, *odd*, or *neither* algebraically.

1. Substitute  $-x$  for each  $x$  in the function.
2. Simplify.
3. Is the function the same as what you started? \_\_\_\_\_
4. Or, did all the signs of the function change? \_\_\_\_\_
5. If you said yes for #3, then  $f$  is even, if you said yes for #4, then  $f$  is odd.  
If you said no for both #3 and #4 then  $f$  is neither even nor odd.

**Example 4.** Determine whether the function  $f(x) = x^3 - 4x$  is *even*, *odd*, or *neither* algebraically.

1. Substitute  $-x$  for each  $x$  in the function.
2. Simplify.
3. Is the function the same as what you started? \_\_\_\_\_
4. Or, did all the signs of the function change? \_\_\_\_\_
5. If you said yes for #3, then  $f$  is even, if you said yes for #4, then  $f$  is odd.  
If you said no for both #3 and #4 then  $f$  is neither even nor odd.

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**Example 5.** Determine whether the function  $f(x) = x^4 - 4x^3$  is *even*, *odd*, or *neither* algebraically.

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**Exercises.** Complete the following on a separate sheet of paper.

Determine whether each function is *even*, *odd*, or *neither* algebraically. Be sure to show all work to support your conclusion. No work — No credit.

1.  $f(x) = x^4 + x^3 + x^2 + x + 1$

2.  $g(x) = x^5 - x^3 - x$

3.  $h(x) = x^6 - x^4 + x^2$

4.  $f(x) = \frac{1}{x^4 + x^2}$

5.  $g(x) = \frac{1}{x^3 + x}$

6.  $f(x) = \frac{1}{x^3 + x^2}$