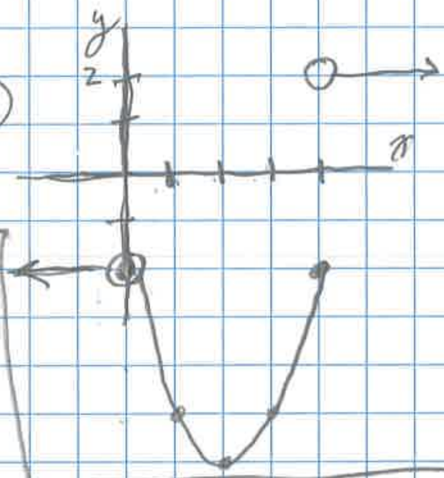
AP Calculus AB — Chapter 3 Review — Fall 2018

1) $f(x) = \begin{cases} -2 & , x < 0 \\ x^2 - 4x - 2 & , 0 \le x \le 4 \\ 2 & , x > 4 \end{cases}$ $(0,-2), (4,-2)$



2) $f(x) = \sin x + \cos x$ on $[-\pi, \pi]$

$$\text{Avg Rate} = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{(\sin \pi + \cos \pi) - (\sin(-\pi) + \cos(-\pi))}{\pi - -\pi}$$

$$= \frac{(0 + -1) - (0 + -1)}{2\pi} = \frac{-1 + 1}{2\pi} = \boxed{0}$$

3) $f(x) = \sin x + \cos x$ at $x = \frac{\pi}{2}$

$$f'(x) = \cos x - \sin x$$

$$f'\left(\tfrac{\pi}{2}\right) = \cos \tfrac{\pi}{2} - \sin \tfrac{\pi}{2} = 0 - 1 = \boxed{-1}$$

4)

$$\lim_{x \to 0} \frac{\frac{1}{x+8} - \frac{1}{8}}{x} = \lim_{x \to 0} \frac{\frac{8 - (x+8)}{8(x+8)}}{\frac{x}{1}} = \lim_{x \to 0} \left[\frac{-x}{8(x+8)} \cdot \frac{1}{x} \right]$$

$$= \lim_{x \to 0} \frac{-1}{8(x+8)} = \frac{-1}{8(0+8)} = \boxed{-\frac{1}{64}}$$

5) Horizontal tangents of $f(x) = 4x^3 + x^2 - 2x + 3$

$$f'(x) = 12x^2 + 2x - 2$$
$$0 = 2(6x^2 + x - 1)$$
$$0 = 2(3x - 1)(2x + 1)$$
$$x = \tfrac{1}{3} \text{ or } x = -\tfrac{1}{2}$$

$$\boxed{\left(\tfrac{1}{3}, \tfrac{70}{27}\right) \quad \left(-\tfrac{1}{2}, \tfrac{17}{4}\right)}$$

$$f\left(\tfrac{1}{3}\right) = 4\left(\tfrac{1}{3}\right)^3 + \left(\tfrac{1}{3}\right)^2 - 2\left(\tfrac{1}{3}\right) + 3$$
$$= \frac{4}{27} + \frac{1}{9} - \frac{2}{3} + 3$$
$$= \frac{4}{27} + \frac{3}{27} - \frac{18}{27} + \frac{81}{27} = \frac{70}{27}$$

$$f\left(-\tfrac{1}{2}\right) = 4\left(-\tfrac{1}{2}\right)^3 + \left(-\tfrac{1}{2}\right)^2 - 2\left(-\tfrac{1}{2}\right) + 3$$
$$= -\frac{4}{8} + \frac{1}{4} + 1 + 3$$
$$= -\frac{2}{4} + \frac{1}{4} + 4 = 3\tfrac{3}{4} = \frac{15}{4}$$

6) Horizontal tangents of $f(x) = 9x^4 - 12x^2 + 4$

$f'(x) = 36x^3 - 24x$

$0 = 12x(3x^2 - 2)$

$12x = 0$ OR $3x^2 - 2 = 0$

$\boxed{x = 0}$ OR $x = \pm\sqrt{\frac{2}{3}}$ → $x = \pm \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{3}$

$x = \pm \frac{\sqrt{6}}{3}$

$\boxed{x = \pm \frac{\sqrt{6}}{3}}$

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7) $\lim_{x \to 0} \frac{-2x^{-1} - 5x^{-3}}{4x^{-4}} = \lim_{x \to 0} \frac{\frac{x^2}{x^2} \cdot \frac{-2}{x} - \frac{5}{x^3}}{\frac{4}{x^4}} = \lim_{x \to 0} \frac{\frac{2x^2 - 5}{x^3}}{\frac{4}{x^4}}$

$= \lim_{x \to 0} \left[\frac{2x^2 - 5}{\cancel{x^3}} \cdot \frac{\cancel{x^4}}{4} \right] = \lim_{x \to 0} \left[\frac{x(2x^2 - 5)}{4} \right]$

$= \frac{0(2(0)^2 - 5)}{4} = \boxed{0}$

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8) $s = 24t - 0.8t^2$ s = meters, t = seconds

a) $v(t) = s'(t) = 24 - 1.6t$

$v(10) = 24 - 1.6(10) = \boxed{8 \text{ m/sec}}$

b) Highest point when $v(t) = 0$

$24 - 1.6t = 0$

$-1.6t = -24$

$\boxed{t = 15 \text{ sec}}$

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9) A) $y = \ln(x)^2$

$(-\infty, 0) \cup (0, \infty)$

B) $y = \sqrt{\frac{1-x}{1+x^2}}$ $1 - x \to 1 - x \geq 0 \Rightarrow 1 \geq x \Rightarrow x \leq 1$

$1 + x^2 \Rightarrow$ Always positive because of x^2

$\boxed{x \leq 1 \text{ or } (-\infty, 1]}$

c) $y = (2x - 7)^{-1}(x + 5)$

$y = \frac{x+5}{2x-7}$ $2x - 7 \neq 0$

$x \neq \frac{7}{2}$

$\boxed{(-\infty, \frac{7}{2}) \cup (\frac{7}{2}, \infty)}$

10) $A = \frac{\sqrt{3}}{2} s^2$, Instantaneous Rate of change when $x = 10$

$s = cm \quad A = cm^2$

$\frac{dA}{ds} = \frac{2\sqrt{3}}{2} s = s\sqrt{3}$ $\quad \left.\frac{dA}{ds}\right|_{x=10} s(\sqrt{3}) = \boxed{10\sqrt{3}\ \frac{cm^2}{cm}}$

11) $y = 2\sqrt{x}$ $\quad x = 9$ $\quad y(9) = 2\sqrt{9} = 2 \cdot 3 = 6$ $\quad (9, 6)$

A) $y = 2x^{\frac{1}{2}} \Rightarrow y' = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$ $\quad y - 6 = \frac{1}{3}(x - 9)$

Tangent $\quad y'(9) = \frac{1}{\sqrt{9}} = \frac{1}{3}$ $\quad y - 6 = \frac{1}{3}x - 3$

$\boxed{y = \frac{1}{3}x + 3}$

B) Normal $\quad m = -3$ $\quad y - 6 = -3(x - 9)$ $\quad \boxed{y = -3x + 33}$

$y - 6 = -3x + 27$

12) $s(t) = (t-2)^2(t-4) = (t^2 - 4t + 4)(t - 4)$

$s(t) = t^3 - 4t^2 + 4t - 4t^2 + 16t - 16$

$s(t) = t^3 - 8t^2 + 20t - 16$

$v(t) = s'(t) = 3t^2 - 16t + 20$ $\quad a(t) = v'(t) = 6t - 16$

$v(t) = (3t - 10)(t - 2)$ $\quad a(\frac{10}{3}) = \cancel{6}^{2}(\frac{10}{\cancel{3}}) - 16 = 20 - 16$

$0 = (3t - 10)(t - 2)$ $\quad = \boxed{4\ m/sec^2}$

$t = \frac{10}{3}$ or $t = 2$

$a(2) = 6(2) - 16 = \boxed{-4\ m/sec^2}$

13) $u(0) = 5$, $u'(0) = -3$, $v(0) = -1$, $v'(0) = 2$

A) $\frac{d}{dv}(uv)$ $\quad uv' + vu'$ $\quad$ B) $\frac{d}{dv}\left(\frac{u}{v}\right)$ $\quad \frac{vu' - uv'}{v^2}$

$= 5(2) + (-1)(-3)$ $\quad = \frac{(-1)(-3) - (5)(2)}{(-1)^2} = \frac{3 - 10}{1} = \boxed{-7}$

$= \boxed{13}$

14) $s = 4 - t^2 \sin t$ $\quad uv' + vu'$

$u = -t^2 \quad v = \sin t$

$u' = -2t \quad v' = \cos t$

$\boxed{\frac{ds}{dt} = -t^2 \cos t - 2t \sin t}$

15) A) $\lim_{x\to 0^-} f(x) = 0$ B) $\lim_{x\to 1^-} f(x) = 1$ C) $\lim_{x\to 3^-} f(x) = 1$

$\lim_{x\to 0^+} f(x) = 1$ $\lim_{x\to 1^+} f(x) = 1$ $\lim_{x\to 3^+} f(x) = \text{DNE}$ (Outside Domain)

$\Rightarrow \lim_{x\to 0} f(x) = \text{DNE}$ $\Rightarrow \lim_{x\to 0} f(x) = 1$ $\Rightarrow \lim_{x\to 3} f(x) = \text{DNE}$

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16) Acceleration is zero when $v'(t)=0$ (zero slopes)
This occurs on the intervals $(0,1)$, $(2,4)$ and $(7,9]$

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17) $s(t) = \tan t + \frac{8}{3}\cos t$ Find $v(\frac{\pi}{6})$.

$v(t) = s'(t) = \sec^2 t - \frac{8}{3}\sin t$

$v(t) = \left(\sec\frac{\pi}{6}\right)^2 - \frac{8}{3}\sin\frac{\pi}{6}$

$v(t) = \left(\frac{2\sqrt{3}}{3}\right)^2 - \frac{\cancel{8}4}{3}\left(\frac{1}{\cancel{2}}\right) = \frac{4\cdot\cancel{3}}{3\cancel{9}} - \frac{4}{3} = \boxed{0}$

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18) $y = \frac{x^3+2}{2x^3+5}$ $\frac{vu' - uv'}{v^2}$ $u = x^3+2$ $v = 2x^3+5$

$u' = 3x^2$ $v' = 6x^2$

$\frac{dy}{dx} = \frac{(2x^3+5)3x^2 - (x^3+2)(6x^2)}{(2x^3+5)^2} = \frac{\cancel{6x^5} + 15x^2 - \cancel{6x^5} - 12x^2}{(2x^3+5)^2}$

$= \boxed{\frac{3x^2}{(2x^3+5)^2}}$

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19) corner cusp vertical tangent

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20) Avg Rate $f(x) = \frac{1}{2}e^x$ on $[0,2]$

$\text{Avg} = \frac{f(b)-f(a)}{b-a} = \frac{\frac{1}{2}e^2 - \frac{1}{2}e^0}{2-0} = \frac{\frac{1}{2}e^2 - \frac{1}{2}e^0}{2} = \frac{\frac{1}{2}e^2 - \frac{1}{2}}{2}$

$= \frac{\frac{1}{2}(e^2-1)}{2} = \frac{e^2-1}{2\cdot 2} = \boxed{\frac{e^2-1}{4}}$