Indefinite Integrals

The previous chapter was strictly on definite integrals, integration with limits of integration. Indefinite integrals do not have limits; but the antiderivative of the function is still taken.

The **set** of all antiderivatives of a function $f(x)$ is the **Indefinite Integral of f with respect to x** and is denoted as

$$\int f(x)\,dx$$

Remember, all antiderivatives differ by a constant, so if $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + C$, where C is the constant of integration. The following table gives a list of rules you should already be familiar with.

Integral Formulas

1. Power Rule for $n \neq -1$: $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$
2. Rule for $n = -1$: $\int \dfrac{1}{x}\,dx = \ln x + C$
3. $\int e^{kx}\,dx = \dfrac{e^{kx}}{k} + C$
4. $\int \sin(kx)\,dx = \dfrac{-\cos(kx)}{k} + C$
5. $\int \cos(kx)\,dx = \dfrac{\sin(kx)}{k} + C$
6. $\int \sec^2 x\,dx = \tan x + C$
7. $\int \csc^2 x\,dx = -\cot x + C$
8. $\int \sec x \tan x\,dx = \sec x + C$
9. $\int \csc x \cot x\,dx = -\csc x + C$
10. $\int \dfrac{dx}{x^2+1} = \tan^{-1}(x) + C$
11. $\int \dfrac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + C$
12. $\int a^x dx = \dfrac{a^x}{\ln a} + C$, where a is a constant

Example 1. Evaluate each integral.

A. $\int \left(-x^3 + \sqrt[3]{x} - 1 + e^{3x}\right) dx$

B. $\int (3\sin x - \sin 3x)\,dx$

Differential Equations

A **Differential Equation** is an equation that is a derivative. Just like in Algebra, when you solve an equation, you use an inverse operation. To "undo" a derivative we take an __________________________________.

Remember, a function can have many antiderivatives, all of which vary by a _________________________.

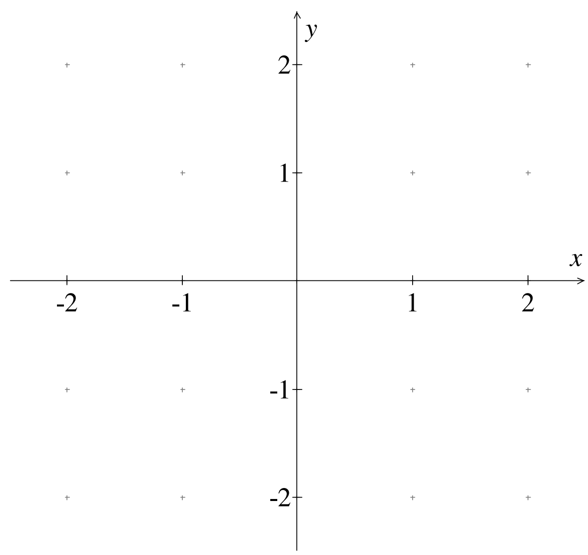
Solving a differential equation involves finding a unique equation that satisfies some ***initial conditions*** or ***initial values***. The **order** of a differential equation is the order of the highest derivative involved in the equation.

Example 2. Solve $\frac{dy}{dx} = \sin x$ by **Separation of Variables** is $y(0) = 2$.

A Graphical Look at Differential Equations

A **Slope Field** (or direction Field) for the first order differential equation $\frac{dy}{dx} = f(x, y)$ is a plot of short line segments with slope $f(x, y)$ for a set of points (x, y) in the plane.

Example 3. On the diagram below, plot the slope field of the differential equation $\frac{dy}{dx} = 2y$.



Example 4. Suppose that you know that the point $(0, -1)$ is a particular solution of the differential equation above. By following slopes, draw on the diagram above what you think the particular solution looks like. (Note: The graph should follow the pattern of the slope field, but may go between the points rather than through them.)

Example 5. Solve the differential equation $\frac{dy}{dx} = 2y$ from Example 3 by first separating the variables. Find the particular solution that contains the point given in Example 4. How does your solution compare to the graph of the slope field in Example 3?