The Fundamental Theorem of Calculus has two parts. These two parts tie together the concept of integration and differentiation and is regarded by some to by the most important computational discovery in the history of mathematics.

Example 1. In section 6.2 we found an estimate for the distance traveled by finding the area between a velocity function and the *x*-axis. If $v(t)$ is the velocity function (that is above the *x*-axis) and the time is from 0 to 10 seconds, how might we use calculus to define the distance traveled?

Example 2. Suppose a car's position is given by $s(t)=\frac{3}{2}t^2+30t+25$ where t is time in seconds, and $0\le t\le 10$.

A. What is the position of the car at $t = 0$ seconds?

B. What is the position of the car at $t = 10$ seconds?

C. What is the change in position of the car from $t = 0$ to time $t = 10$ seconds?

D. How does this question relate to the previous example?

The Fundamental Theorem of Calculus – Part 2, The Evaluation Part

If f is continuous at every point of $[a, b]$, then $\int_a^b f(x)\,dx = F(b)-F(a)$, where ***F*(*x*) is an antiderivative of *f*(*x*)**. This last phrase is the toughest part to understand.

The rules below is a list of antiderivatives you should become familiar with. These rules are stated in the next chapter, but what you know about derivatives, you should be able to make these connections.

Integral Formulas

1. Power Rule for x^n when $n \neq -1$: $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$

2. Rule for x^n when $n = -1$: $\int \frac{1}{x} \, dx = \ln|x| + C$

3. $\int e^{kx} \, dx = \frac{e^{kx}}{k} + C$

4. $\int \sin(kx) \, dx = \frac{-\cos(kx)}{k} + C$

5. $\int \cos(kx) \, dx = \frac{\sin(kx)}{k} + C$

6. $\int \sec^2(x) \, dx = \tan(x) + C$

7. $\int \csc^2(x) \, dx = -\cot(x) + C$

8. $\int \sec(x)\tan(x) = \sec(x) + C$

9. $\int \csc(x)\cot(x) = -\csc(x) + C$

10. $\int a^x \, dx = \frac{a^x}{\ln a} + C$

Example 3. $\int_0^3 x^2 \, dx =$

Example 4. $\int_{\pi/2}^{\pi} (1 + \cos x) \, dx =$

Example 5. $\int_{-1}^{2} 3^x \, dx =$

Example 6. $\int_4^9 f'(x) \, dx =$

Example 6 will be an extremely important concept throughout the rest of the semester.

Using the **Evaluation Part** of the **Fundamental Theorem of Calculus**, we are going to develop the concept of the **Antiderivative Part** of the **Fundamental Theorem of Calculus**. Our goal here isn't really to prove the **Fundamental Theorem of Calculus**, the **Antiderivative Part**, but to understand how it works.

Here is a quick overview:

1. We are going to create a function that is defined as an integral, then,
2. Using this function, we are going to find the derivative of this function; thus, tying the two concepts of calculus together forever.

Keep in mind that if we can define a function as an integral and take a derivative, then we can answer all the same types of questions about increasing, decreasing, concave up, concave down, and inflection points that we did earlier in the year; you haven't forgotten all those reasons, have you?

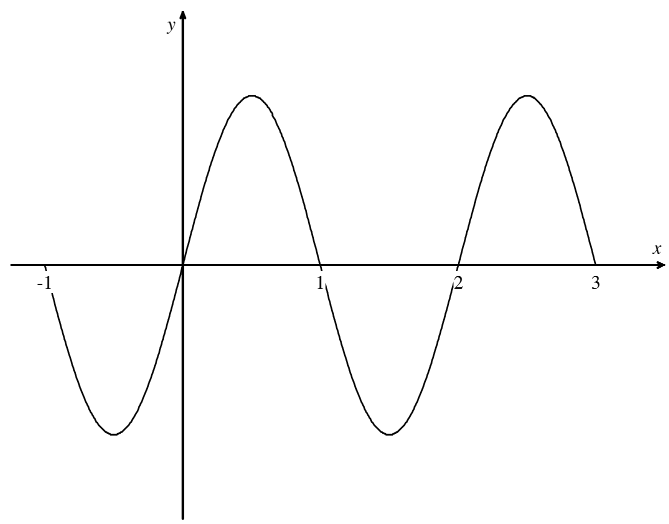
Step #1: So, to see how it is possible to define a function using an integral, consider the examples below.

The graph of $f(t)$ given below has odd symmetry and is periodic (with period = 2). Also $\int_0^1 f(t)\,dt = \frac{4}{3}$.

Example 7. Let $F(x) = \int_0^x f(t)\,dt$.

Complete the following table.

x	$F(x)$
-1	
0	
1	
2	
3	

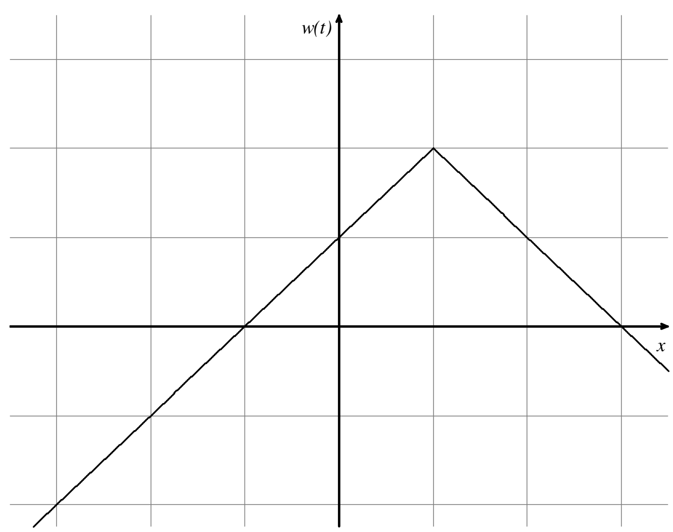


Example 8. Let $g(x) = \int_{-2}^x w(t)\,dt$, where the Graph of $w(t)$ is given below.

A. Find $g(0)$.

B. Find $g(2)$.

C. Find $g(-3)$.



Now that a function is defined as an integral, let us see how to find a derivative of such a function. While interpreting a function defined as an integral is a valid skill in its own right, our goal here is to simply discover patterns found when taking the derivative.

In Example 6, $\int_4^9 f'(x)\,dx = f(9) - f(4)$, where $f(x)$ is the antiderivative of $f'(x)$.

Example 9. Find $\int_a^x h'(t)\,dt$, where a is a constant.

Example 10. Now, find $\frac{d}{dx}\int_a^x h'(t)\,dt$.

The Fundamental Theorem of Calculus, Antiderivative Part (Simple)

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Example 11. If $g(x) = \int_{-2}^x w(t)\,dt$, then $g'(x) = ?$

Example 12. $\frac{d}{dx}\left[\int_3^x \left(5t^2 - 6t + 1\right)dt\right] =$

The next example has limits on the integral that are functions of *x*, as opposed to simply limit of *x* and an upper limit of a constant.

Example 13. $\frac{d}{dx}\left[\int_{v(x)}^{u(x)} h'(t)\,dt\right]$

The Fundamental Theorem of Calculus, Antiderivative Part (Extended)

$$\frac{d}{dx}\left[\int_{v(x)}^{u(x)} f(t)\,dt\right] = f\left(u(x)\right)\bullet u'(x) - f\left(v(x)\right)\bullet v'(x)$$

Example 14. Find $\frac{d}{dx}\left[\int_{x^2}^{3x} f(t)\,dt\right]$

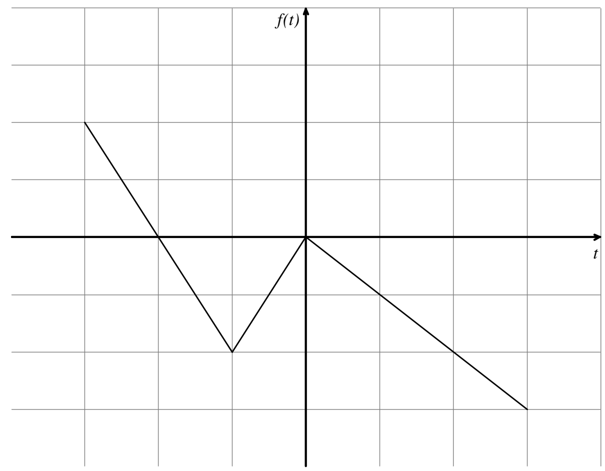
Example 15. Let $g(x)=\int_{5x}^{3x^2}\sqrt{1+t^3}\,dt$. Find $g'(x)$.

Putting It All Together.

Example 20. Suppose the functions is the graph of $f(t)$ and $g(x)=\int_{-1}^{x} f(t)\,dt$.

A. Complete the table:

x	$g(x)$
–3	
–2	
–1	
0	
1	
2	
3	



B. What are the intervals on which g is increasing or decreasing? Justify each response.

C. What are the intervals on which g is concave up or concave down? Justify each response.

D. For what value of x does g have a relative maximum? Justify your response.

E. For what value of x does g have an inflection point? Justify your response.

F. Graph $g(x)$.