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**Example 1.** Suppose from the 3<sup>rd</sup> to 5<sup>th</sup> hour of a recent road trip, you set the speed control to exactly 70 miles per hour. How far have you traveled during this time?

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**Example 2.** Sketch a graph modeling the situation from Example 1. Geometrically, how can the total distance traveled be determined?

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**Example 3.** What if velocity is NOT constant. Suppose velocity can be given by the function  $v(t) = 10t - t^2$ , where  $t$  is in hours and  $v(t)$  is in miles per hour, and we want to know the total distance traveled during the first 10 hours. Sketch this graph below. Can the total distance traveled be found geometrically the same way as before? Why or why not?

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What we are ready to explore is an extremely important Calculus concept; finding the area under a curve. The method used to find the total distance traveled in Example 3 is similar to the method used in Example 2. But the key to finding total distance in Example 3 is to break the time intervals into short segments, so that the velocity over these time segments is almost constant (this will require quite a few intervals). If the velocity is almost constant for each time interval, then the distance traveled for each time interval can be found (which is just the area of an extremely thin rectangle) and add all the areas of all the rectangles together. This concept sounds simple and involves an extremely important Calculus concept.

We will spend MUCH more time with this later, but it turns out that if we are given the graph of a rate of change (like velocity in mile per hour) we will be able to find the total accumulated change over an interval (like total distance traveled, in miles) by finding the area under the curve.

Any of this may not make sense to you, but the goal for now is to figure out a way to find the *area under the curve*. This chapter discusses 5 ways to *approximate* this area, *LRAM*, *RRAM*, *MRAM*, *Trapezoid Rule*, and *Simpson's Rule*. We are going to learn 4 of these, all but Simpson's Rule as it is not the AP Calculus AB curriculum.

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**The Rectangular Approximation Method (RAM)**, also known as *Riemann Sums*.

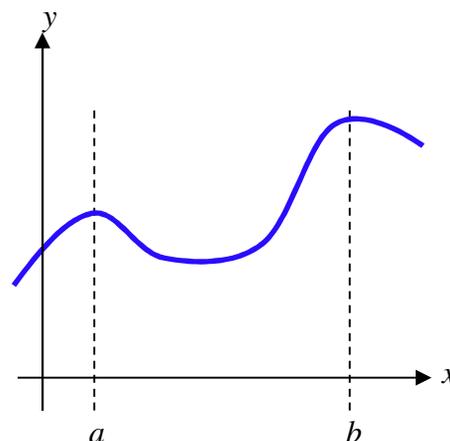
Suppose we want to know the area of the region bounded by a curve, the  $x$ -axis, and the lines  $x = a$  and  $x = b$ , as shown at the right.

First, divide the interval from  $a$  to  $b$  into subintervals. The example below shows 4 and 8 subintervals, respectively.

Next, draw rectangles using the width of each subinterval as the base.

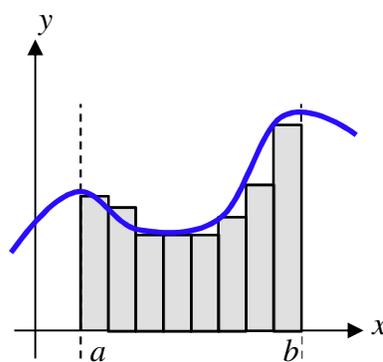
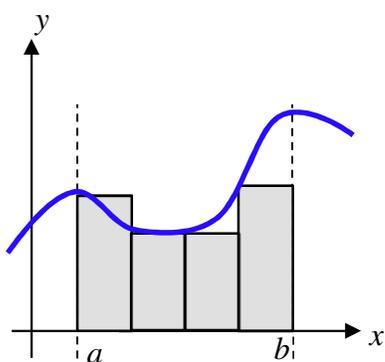
The height of each rectangle is determined by the function value at a point in the specific subinterval and can be determined using 3 different methods.

We could use the left endpoint of each subinterval (called LRAM), the right endpoint (RRAM), or the midpoint of each subinterval (MRAM).



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**Example 4.** Which method is shown in the two graphs below?



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**Example 5.** The total area under the curve is approximately equal to the total area of all the rectangles. Which graphs in Example 4 gives a better approximation of the area under the curve? Why? How could the approximation be improved?

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**Summary of the Process:**

Step 1: A sketch is almost mandatory.

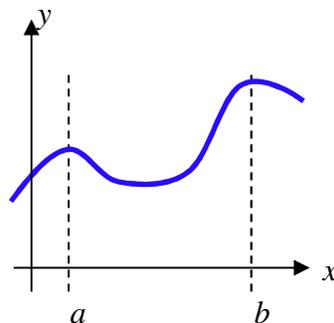
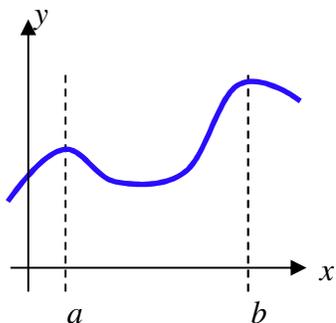
Step 2: Divide (or Partition) the interval into  $n$  subintervals.

Step 3: Create  $n$  rectangles whose base equals the width of each subinterval and whose height is determined by the function value at the left endpoint, the right endpoint, or the midpoint of the subinterval.

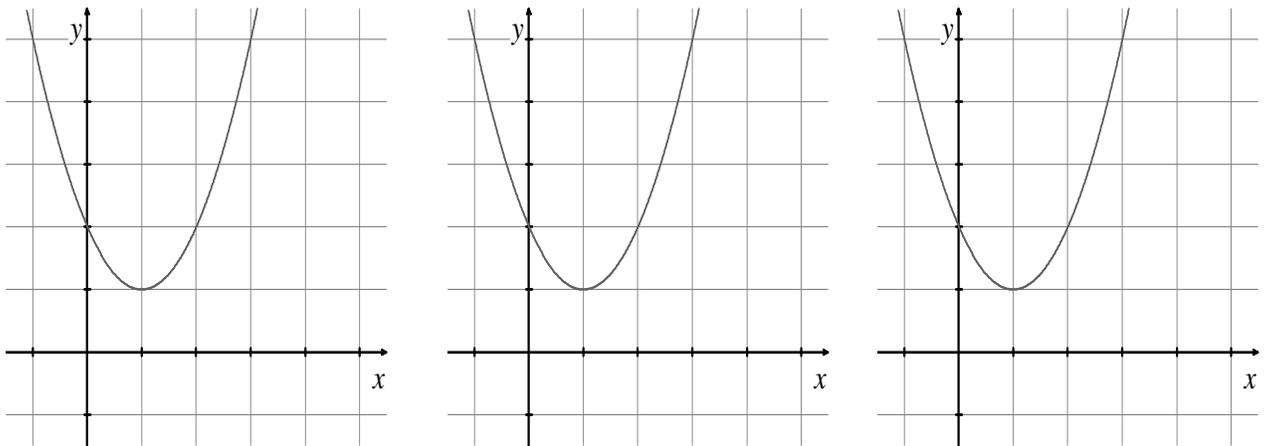
Step 4: Find the area of all  $n$  rectangles and add them all together.

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**Example 6.** Using 4 rectangles, illustrate the use of RRAM and MRAM on the graphs below.



**Example 7.** Use 4 rectangles to approximate the area under the graph of  $y = x^2 - 2x + 2$  from  $x = 1$  to  $x = 3$ . Use LRAM, RRAM, and then MRAM.



**Example 8.** Using your rectangles as a guide, find each approximation.

A. LRAM =

B. RRAM =

C. MRAM =

**Example 9.** It is not necessary to have a graph to estimate the area. Suppose the table below shows velocity of a model train engine moving along a track for 10 seconds.

<b>Time (sec)</b>	0	1	2	3	4	5	6	7	8	9	10
<b>Velocity (in./sec)</b>	0	12	22	10	5	13	11	6	2	6	0

A. Using Left Riemann Sum with 10 subintervals, estimate the distance traveled by the engine in the first 10 seconds.

B. Using Midpoint Riemann Sum with 5 subintervals, estimate the distance traveled by the engine in the first 10 seconds.

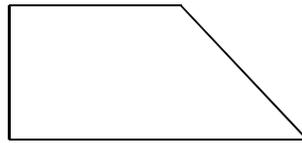
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## The Trapezoidal Rule (Section 6.5)

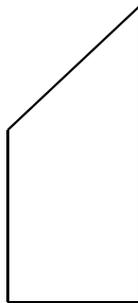
While rectangles make a fairly good approximation, it's easy to see that we're going to need a lot of them to provide a good estimate. We can find a better estimate in less time if we use trapezoids. If we were to partition the interval into subintervals like we did before, we can use each subinterval to create a trapezoid if we just connect the function values of the left and right endpoints. Before we begin, let's make sure you understand the area formula for a trapezoid.

$$\text{Area of a Trapezoid: } A = \frac{1}{2} \cdot h \cdot (b_1 + b_2)$$

While not all trapezoids must look like this, the one's we're going to be using will, so we'll stick with this picture. Label all the parts of the area formula on the picture below.



The biggest difference will be the orientation of the trapezoid. The ones we are going to be drawing will look like:



Draw a set of axes on the picture above and a function that goes through the top left and top right points of the trapezoid. The "height" of the trapezoid is just the width of a subinterval, and the "bases" are going to be the function values of the left and right endpoints.

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**Example 10.** Use 4 trapezoids to approximate the area under the curve of  $y = x^2 - 2x + 2$  from  $x = 1$  to  $x = 3$ . Sketch the trapezoids. The Trapezoidal Rule connects the left and right endpoints with a segment. This method of approximation turns out to be pretty good.

