

Name: \_\_\_\_\_ Per: \_\_\_\_\_ Date: \_\_\_\_\_

To complete these notes, go to [www.BishSoft.org](http://www.BishSoft.org) and watch the video for section 5.1.

Extrema (plural for extremum) are the maximum or minimum values of functions. We need to distinguish between absolute extrema and relative extrema, and how to locate them. Not only do you need to your calculator to calculate a maximum or minimum value, but you must also use calculus to justify how maximum and minimum values.

### Definition of Absolute Extreme Values (The BIGGEST or smallest $y$ -value in the interval.)

Let  $f$  be a function with domain  $D$ . Then  $f(c)$  is the

(A) **absolute maximum value** on  $D$  if and only if  $f(c) \geq f(x)$  for all  $x$  in  $D$ .

(B) **absolute minimum value** on  $D$  if and only if  $f(c) \leq f(x)$  for all  $x$  in  $D$ .

### The Extreme Value Theorem

If  $f$  is continuous on a closed interval  $[a, b]$ , then  $f$  has both a maximum value and a minimum value on the interval.

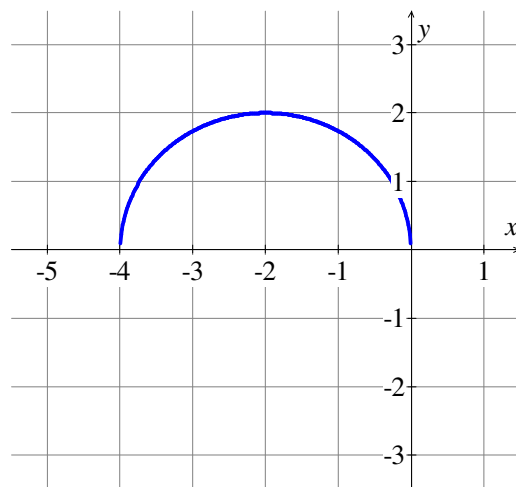
**Example 1.** Draw a sketch and find the absolute extrema of the function  $f(x) = \sqrt{4 - (x+2)^2}$  on each given the interval. If no maximum or minimum exists, which part of the extreme value theorem hypothesis isn't satisfied?

A.  $[-4, 0]$

B.  $[-2, 0]$

C.  $(-4, -2)$

D.  $[1, 2)$



### Relative Extrema and Critical Numbers

#### Definition of Local (Relative) Extreme Values

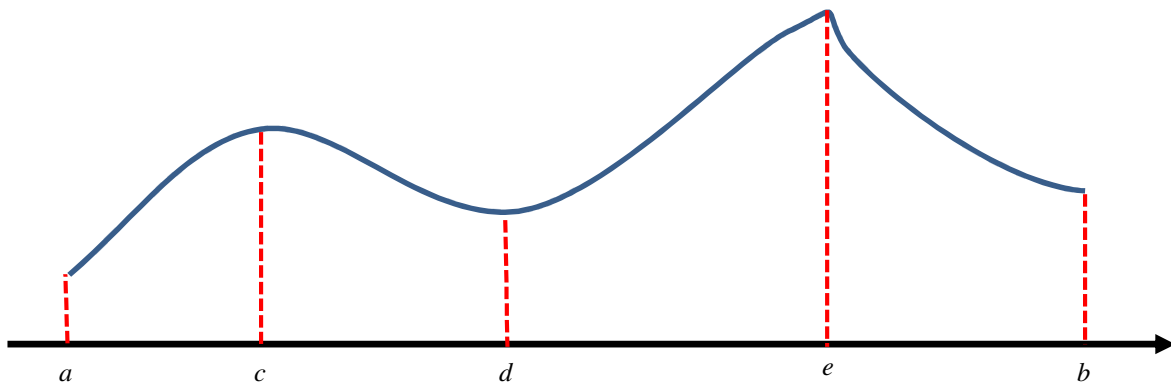
Let  $c$  be an interior point of the domain of the function  $f$ . Then  $f(c)$  is a

(A) **local maximum value** at  $c$  if and only if  $f(c) \geq f(x)$  for all  $x$  in some open interval containing  $c$ .

(B) **local minimum value** at  $c$  if and only if  $f(c) \leq f(x)$  for all  $x$  in some open interval containing  $c$ .

Basically relative extrema exist when the value of the function is larger (or smaller) than all other function values relatively close to that value.

**Example 2.** Suppose you were given the function below. Label  $f(a)$  through  $f(e)$  as absolute or relative extrema.



When given a graph it is fairly simple to identify the extrema. The question to be asked then is, how do we find the extrema when we do not have a graph given to us?

**Example 3.** Except at the endpoints  $a$  and  $b$ , what do you notice about the derivative at the relative extrema in the last example?

### Definition of a Critical Point

Let  $f$  be defined at  $c$ . If  $f'(c) = 0$  or is undefined at  $c$ , then  $c$  is a **Critical Point** of  $f$ .

### Relative Extrema Occur Only at Critical Points

If  $f$  has a relative minimum or relative maximum at  $x = c$ , then  $c$  is a critical number of  $f$ .

**IMPORTANT:** Just because the derivative is equal to zero (or undefined) does not mean there is a relative maximum or minimum there. **Relative Extrema** can occur **ONLY** at critical points, and critical points occur **ONLY** when the derivative is either 0 or undefined, however, it is possible for the derivative to equal zero (or undefined) and there be **NO** extrema there.

If you need to convince yourself of this, try  $f(x) = x^3$  and  $f(x) = \sqrt[3]{x}$  at  $x = 0$ . Is the derivative zero? Undefined? Is there a maximum or a minimum?

### Guidelines for Finding Absolute/Relative Extrema on a Closed Interval

1. Find the critical numbers of  $f$  in  $(a, b)$ . Do this by \_\_\_\_\_.  
These critical numbers AND the endpoints make up a list of **Candidates** for the extrema.
2. Evaluate each **Candidate** by plugging these numbers into the original function.
3. Evaluate  $f$  at the endpoints of  $[a, b]$ . **DON'T FORGET THIS PART IF IT THE INTERVAL IS CLOSED!!!!**
4. The least of the values from steps 2 and 3 is the absolute minimum, and the greatest of these values is the absolute maximum.  
If the interval is closed and the endpoints do no result in an absolute max or min, a sign chart can be used to determine whether or not the endpoints result in a relative max or min.

**Example 4.** The critical numbers (or critical points) are \_\_\_\_ values, while the maximums/minimums of the function are \_\_\_\_ values. In other words, if the point  $(2, 70)$  is a relative minimum, then the minimum of the function is \_\_\_\_\_, and it occurs at \_\_\_\_\_. (This is how you correctly describe extrema on any test.)

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**Example 5.** Find the extrema of  $f(x) = 3x^4 - 4x^3$  on the interval  $[-1, 2]$ .

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**Example 6.** Find the extrema of  $g(x) = 2x - 3x^{2/3}$  on the interval  $[-1, 3]$ . Use your graphing calculator to investigate first.

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## Optimization – Section 5.4

“Optimization” problems and problems we have done so far is that the equation must be written by you.

Remember, **if you have a closed interval you MUST test the endpoints of the function as well.**

### Steps for Solving Applied Optimization Problems

- Step 1:** Understand the problem. Read it carefully, and ask yourself, “Self, what is the quantity to be maximized or minimized? What are the quantities which it depends on?”
- Step 2:** Draw a diagram if possible. Sometimes, more than one diagram helps you to determine how all the quantities are related. Identify all quantities from step 1 on your diagram.
- Step 3:** Assign variables to the quantities from step 1.
- Step 4:** Determine a function for the quantity to be optimized.
- Step 5:** Eliminate all but ONE variable and determine the domain of the resulting function. If this is necessary, you will need a second equation that relates the variables in the original equation.
- Step 6:** Optimize the function. (Find the absolute MAX or MIN)
- (A) Calculate the derivative and find the critical numbers.
  - (B) If the domain is a closed interval, compare the function’s value of the critical numbers with that of the endpoints.
  - (C) If the domain is an open interval (or infinite interval), use the **First or Second Derivative Tests** to analyze the behavior of the function. (This will be explained in the next few sections).

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**Example 7.** A rectangle is bounded by the  $x$ -axis and the semicircle  $y = \sqrt{25 - x^2}$ . What length and width should the rectangle have so that its area is a maximum?

