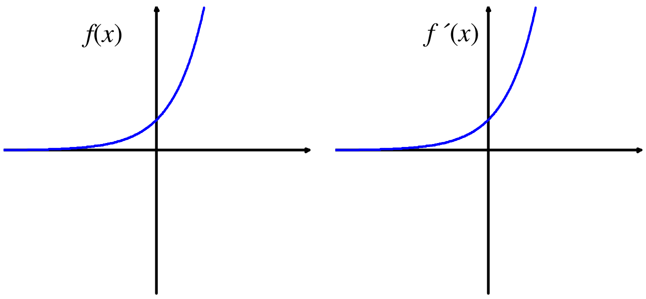
AP Calculus AB — 4.4 Derivatives of Exponential and Logarithmic Functions

Name:____Per:________Date:____________________

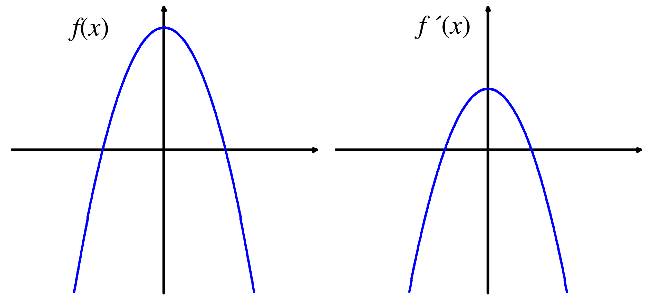
To complete these notes, go to www.BishSoft.org and watch the video for section 4.4.

Example 1. Which of the pairs of graphs could represent the graph of a function **AND** its derivative?

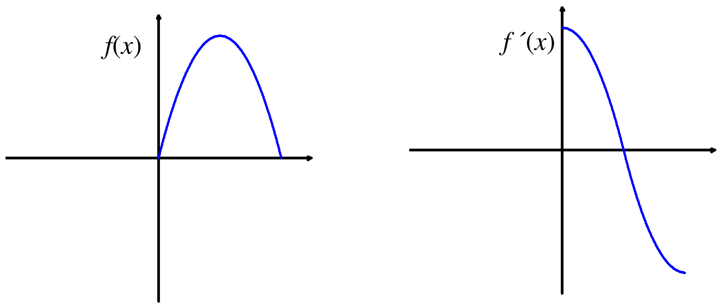
I.



II.



III.



(A) I only **(B)** II only **(C)** III only **(D)** I and III only **(E)** II and III only

Now for the easiest derivative rule of the year. Notice the first pair of graphs in **(I)** above.

Derivative of $f(x) = e^x$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

The proof of this can be done using the definition of a derivative.

The Chain Rule and $f(x) = e^x$

If u is a differentiable function of x, then

$$\frac{d}{dx}\left[e^u\right] = e^u \bullet \frac{du}{dx}$$

Example 2. Find $\frac{d}{dx}\left[e^{2x-1}\right]$.

Example 3. Find $\frac{d}{dx}\left[e^{-3/x}\right]$.

The inverse of an exponential function is the natural logarithm function.

Derivative of $f(x) = \ln x$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Example 4. Prove the derivative rule above using implicit differentiation.

The Chain Rule and $f(x) = \ln x$

If u is a differentiable function of x, then

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \bullet \frac{du}{dx} \quad \ldots\text{or}\ldots \quad \frac{d}{dx}[\ln u] = \frac{u'}{u}$$

Example 5. Let $y = \ln(2x+2)$. Find y'.

Example 6. Let $f(x) = \ln(\tan x)$. Find $f'(x)$.

Logarithmic Differentiation

We can use the properties of logarithms to simplify some problems. Here's a quick refresher on those properties.

Definition of a Logarithm: $\log_b y = x \iff b^x = y$ **Change of Base Formula:** $\log_b a = \frac{\log a}{\log b}$ or $\log_b a = \frac{\ln a}{\ln b}$

Properties of Logarithms:

Product Property: $\log_b(MN) = \log_b(M) + \log_b(N)$

Quotient Property: $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

Power Property: $\log_b(M^k) = k \bullet \log_b(M)$

Example 7. Use the properties of logarithms to rewrite the function, then find the derivative of $y = \log_5 \sqrt{x}$.

Example 8. Use the technique of logarithmic differentiation to find $\frac{dy}{dx}$ for $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$.

By utilizing the rules of logarithms and implicit differentiation, you can turn an exponential equation into an equation involving logarithms that is usually easier to deal with.

Example 9. Find $\frac{dy}{dx}$ if $y = 2^x$.

Example 10. Find $\frac{dy}{dx}$ if $y = 3^x$.

The previous two examples lead us to the following result.

Derivative of a^u, a is a Constant

If $a > 0$ and $a \neq 1$, and u is a differentiable function of x, then

$$\frac{d}{dx}\left[a^u\right] = \ln a \bullet a^u \frac{du}{dx}$$

Example 11. Find the derivative of the function $g(x) = e^{5x} + 7^x + \ln\left(x^3+4\right)$.