Name:_____Per:_____Date:_____

To complete these notes, go to www.BishSoft.org and watch the video for section 4.3.

Derivatives of Inverse Trigonometric Functions

One of the things in the AP Calculus AB Course Description is "Use of implicit differentiation to find the derivative of an inverse function". With that said, let's find the derivative of the inverse sine function using implicit differentiation.

Example 1. Suppose $y = \sin^{-1} x$. Find $\frac{dy}{dx}$ using implicit differentiation.

Derivatives of Inverse Trigonometric Functions where *u* is a function of *x*

1. $\frac{d}{dx}\left[\sin^{-1}(u)\right] = \frac{u'}{\sqrt{1-u^2}}$

2. $\frac{d}{dx}\left[\cos^{-1}(u)\right] = \frac{-u'}{\sqrt{1-u^2}}$

3. $\frac{d}{dx}\left[\tan^{-1}(u)\right] = \frac{u'}{1+u^2}$

4. $\frac{d}{dx}\left[\cot^{-1}(u)\right] = \frac{-u'}{1+u^2}$

5. $\frac{d}{dx}\left[\sec^{-1}(u)\right] = \frac{u'}{|u|\sqrt{u^2-1}}$

6. $\frac{d}{dx}\left[\csc^{-1}(u)\right] = \frac{-u'}{|u|\sqrt{u^2-1}}$

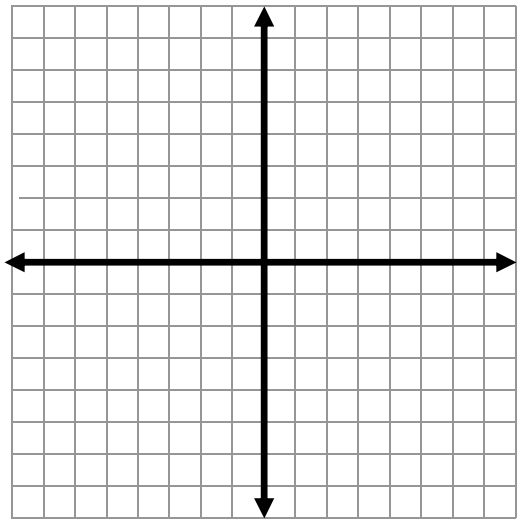
Note: Remember, domains are restricted to make them functions. Also, $\sin^{-1}(x)$ and $\arcsin(x)$ are the same thing. Also, this need to be memorized as quickly as possible.

Example 2. Find the derivative of $f(t) = \sin^{-1}(t^2)$.

Example 3. Find the derivative of $y = \tan^{-1}\left(\sqrt{x-1}\right)$

Derivatives of Other Inverse Functions

Example 4. Graph the line $y = 4x + 1$.



A. What is the slope of the line.

B. Graph the inverse function.

C. What is the slope of the inverse function?

D. If $(2, 9)$ is on the original line, what point does it correspond to on the inverse function?

The slope of the line $(2, 9)$ on the original function is the _____ of the slope of the inverse. The difference is that the slope of the inverse is calculated using the point _____ instead of $(2, 9)$.

Derivative of the Inverse Function at a Point (a, b) – (This implies that (b, a) is on the original function).

If $f^{-1}(a) = b$, that is (a, b) is a point on $f^{-1}(x)$, the **Inverse Function Slope Relationship** relates the derivatives by the equation $\left(f^{-1}\right)'(a) = \dfrac{1}{f'(b)}$.

In other words, the slopes of f and f^{-1} at the inverse points are reciprocals.

Example 5. Let $f(x) = x^5 + 2x - 1$. Verify $(0, -1)$ is on the graph. Find $\left(f^{-1}\right)'(-1)$.

Example 6. Let $f(x) = x^3 + 2x - 1$. Find $\left.\dfrac{df^{-1}}{dx}\right|_2$.