Name:____Per:________Date:____________________

To complete these notes, go to www.BishSoft.org and watch the video for section 4.1.

Suppose you need to differentiate $h(x)\sqrt{x^2+1}$. Up to this point in the course, we have no tools with which to differentiate this function because there is a function, (x^2+1), ***inside*** another function $\sqrt{x}$ (i.e., a composite function). If we were to let

$$f(u)=\sqrt{u}\text{ , and }u=g(x)=x^2+1,$$

then $h(x)=f(g(x))=\sqrt{x^2+1}$. We can differentiate each of these separately, however, we need a rule allowing us to differentiate the original function, or more generally any composite function. Let us consider an example that may shed some light on how this might be accomplished.

Example 1. The length, *L*, in cm, of a steel bar depends on the air temperature, *H* °*C*, and the temperature *H* depends on time, *t*, measured in hours. If the length increases by 2 cm for every degree increase in temperature, and the temperature is increasing at 3 °*C* per hour, how fast is the length of the bar increasing? What are the units for your answer?

This last example illustrates a practical look at how the following theorem actually works.

The Chain Rule

If $y=f(u)$ is a differentiable function of *u* and $u=g(x)$ is a differentiable function of *x*, then $y=f(g(x))$ is a differentiable function of *x* and

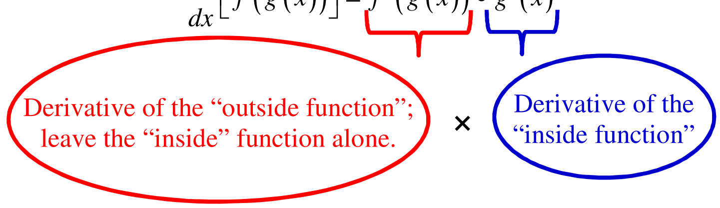
$$\frac{dy}{dx}=\frac{dy}{du}\bullet\frac{du}{dx}$$

of equivalently,

$$\frac{d}{dx}\left[f(g(x))\right]=f'(g(x))g'(x).$$

Composite functions have an "inside function" and an "outside function". Another way to look at this would be

$$\frac{d}{dx}\left[f(g(x))\right]=f'(g(x))\bullet g'(x)$$



The toughest part, at first, is learning to identify the "inside" and "outside" functions.

Example 2. Each of the following examples can be done without using the chain rule. First, state how to find the derivative without using the chain rule, and then use the chain rule to differentiate. State the "inside" and "outside" parts.

A. $f(x) = \dfrac{2}{3x+1}$

B. $g(x) = \left(x^2+2\right)^3$

C. $h(x) = \sin(2x)$

Combining ALL the rules: Power, Product, Quotient, and Chain

Example 3. Find $k'(x)$ if $k(x) = \left(x^2+1\right)\sqrt{2x-3}$.

Example 4. Find $\dfrac{dg}{dt}$ if $g = \left(\dfrac{t-2}{2t+1}\right)^9$

Example 5. For each of the following, use the given information that $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, and $h'(5) = -2$ to find $f'(5)$, if possible. If it is not possible, state what additional information is required.

A. $f(x) = g(x)h(x)$

B. $f(x) = g(h(x))$

C. $f(x) = \dfrac{g(x)}{h(x)}$

D. $f(x) = \left[g(x)\right]^3$