AP Calculus AB — 3.3 Rules for Differentiation

Name:____Per:_________Date:____________________

To complete these notes, go to www.BishSoft.org and watch the video for section 3.3

Up until now, you have been finding derivatives by using the **Definition of a Derivative** or the Alternative Definition, the **Derivative at a Point**. But, many functions have Rules (shortcuts) which can used instead. For functions without Rules, the definition has to be used. So, let's look at some of the rules and how they are used.

Rule 1: Derivative of a Constant Function

If c is any constant value, then $\frac{d}{dx}[c]=0$.

Think about, the graph of any constant function is a horizontal line. The slope of a horizontal line is always 0, so it follows the derivative of a constant is 0.

Example 1. Let $f(x)=5$. Find $f'(x)$.

Rule 2: Power Rule

If n is any number, then $\frac{d}{dx}\left[x^n\right]=n \bullet x^{n-1}$, provided x^{n-1} exists.

Note: In section 3.3, the textbook distinguishes between n being a positive integer (rule 2), n being a negative integer (rule 7) and n being a rational number (rule 9, section 4.2). The distinction is made so that they may prove each separate case in the textbook. However, the use of the power rule is unchanged for all three different values of n.

The ***key*** to using the power rule is to get comfortable using exponent rules to write a function as a power of x.

Example 2. Let $f(x)=x^5$. Find $f'(x)$.

Example 3. Let $f(x)=\sqrt[3]{x^2}$. Find $f'(x)$.

Example 4. Let $f(x)=\frac{1}{x^4}$. Find $f'(x)$.

Rule 2: The Constant Multiple Rule

If u is a differentiable function of x and c is a constant, then $\frac{d}{dx}[cu]=c\frac{du}{dx}$.

Example 5. Let $y=5x^7$. Find $\frac{dy}{dx}$.

Example 6. Let $g(x)=\frac{4}{5x^3}$. Find $g'(x)$.

Rule 4: The Sum and Difference Rule

If *u* and *v* are differentiable functions of *x*, then wherever *u* and *v* are differentiable $\frac{d}{dx}[u \pm v] = \frac{du}{dx} \pm \frac{dv}{dx}$.

Example 7. Let $y = x^3 + 4x^2 - 2x + 7$. Find y'.

Example 8. Let $g(x) = \frac{3}{(-2x)^4} - \frac{x}{2} + \frac{1}{4}$. Find $g'(x)$.

Example 9. Find the equation of the tangent line to the function $f(x) = 4x^3 - 6x + 5$ when $x = -2$.

Example 10. Let $h(x) = (x^2 + 1)(2x - 5)$. Find $h'(x)$.

Example 11. The volume of a cube with sides of length *s* is given by $V = s^3$. Find $\frac{dV}{ds}$ when *s* = 4 centimeters.

Using Rule 4, we know that the derivative of the sum of two functions is the sum of the derivatives of the two functions. **<u>This *does*</u>** ***<u>not work for the product and quotient of two functions</u>***. To illustrate this, we look at the following example.

Example 12. Find $\frac{d}{dx}\left[x^2 \bullet 3x\right]$

Rule 5: The Product Rule

If *u* and *v* are differentiable functions of *x*, then

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}.$$

This is also written as

$$\frac{d}{dx}[uv] = uv' + vu'.$$

For polynomial functions it is not always necessary to use the product rule, however, with trigonometric, exponential, logarithmic, and other functions, it is a necessary tool.

Example 13. Let $y=\left(3+2\sqrt{x}\right)\left(5x^3-7\right)$.

A. Find $\frac{dy}{dx}$ without using the product rule.

B. Find $\frac{dy}{dx}$ using the product rule.

Rule 6: The Quotient Rule

If u and v are differentiable functions of x, and $v \neq 0$,then

$$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}.$$

This is also written as

$$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{vu'-uv'}{v^2}.$$

Example 14. Find $\frac{d}{dx}\left(\frac{x}{x^2+1}\right)$.

Example 15. Find $\frac{d}{dx}\left(\frac{5x^2}{x^3+1}\right)$

Second and Higher Order Derivatives

The first derivative of y with respect to x is denoted y' or $\frac{dy}{dx}$. The second derivative of y with respect to x is denoted y'' or $\frac{d^2y}{dx^2}$.

The second derivative is an example of a higher-order derivative. We can continue to take derivatives (as long as the exist) using the following notation.

First Derivative	y'	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}\left[f(x)\right]$
Second Derivative	y''	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}\left[f(x)\right]$
Third Derivative	y'''	$f'''(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}\left[f(x)\right]$
Fourth Derivative	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}\left[f(x)\right]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
***n*th Derivative**	$y^{(n)}$	$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$\frac{d^n}{dx^n}\left[f(x)\right]$

Example 16. Find $\frac{d^4}{dx^4}\left[-5x^8+2x^6-9x^3+32x-1\right]$

Example 17. Let $f(x)=\frac{x}{x-1}$. Find $f''(x)$.

Example 18. If $f^{(4)}(x)=2\sqrt{x}$, find $f^{(5)}(x)$.