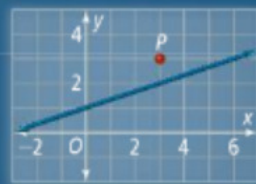
Solve It! Write your solution in the white space on page 81.

Getting Ready!

The equation of the line is $y = \frac{1}{3}x + 1$. How could you change the y-intercept so the graph of a second equation passes through point P? How could you change the slope so that the graph of a third equation also passes through point P? How are the new lines related to the original line?



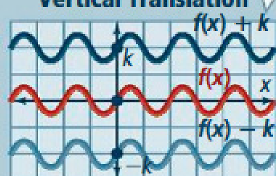
Page 81

A **parent function** is the simplest form in a set of functions that form a family. Each function in the family is a **transformation** of the parent function.

One type of transformation is a **translation**. A translation shifts the graph of the parent function horizontally, vertically, or both without changing shape or orientation. For a positive constant k and a parent function $f(x)$, $f(x) \pm k$ is a vertical translation. For a positive constant h, $f(x \pm h)$ is a horizontal translation.

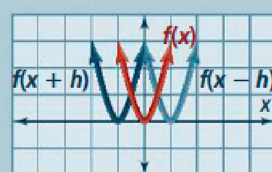
Adding k to the outputs shifts the graph up.

Vertical Translation



Subtracting h from the inputs shifts the graph right.

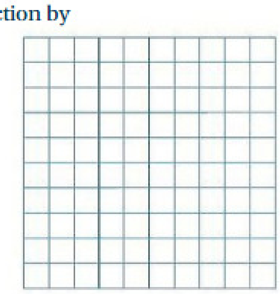
Horizontal Translation



Problem 1 Vertical Translation Page 82

Practice

1. How is the function $y = x - 3$ related to $y = x$? Graph the function by translating the parent function.

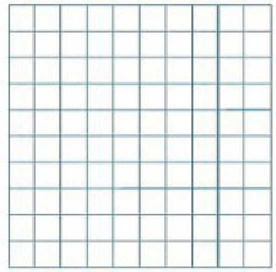


2. Make a table of values for $f(x)$ after translating $f(x)$ 3 units up.

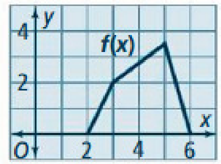
x	$f(x)$
−2	3
0	1
1	−2
3	−1

Problem 2 **Horizontal Translation** Page 83

Practice **3.** For the function $y = (x - 4)^2$, identify the horizontal translation of the parent function $f(x) = x^2$. Then graph the function.

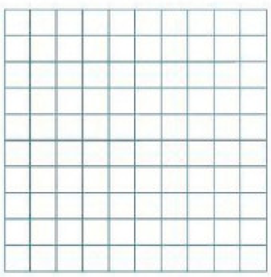


4. The graph of the function $f(x)$ is shown at the right.



a. Make a table of values for $f(x)$ and $f(x + 3)$.

b. Graph $f(x)$ and $f(x + 3)$ on the same coordinate grid.



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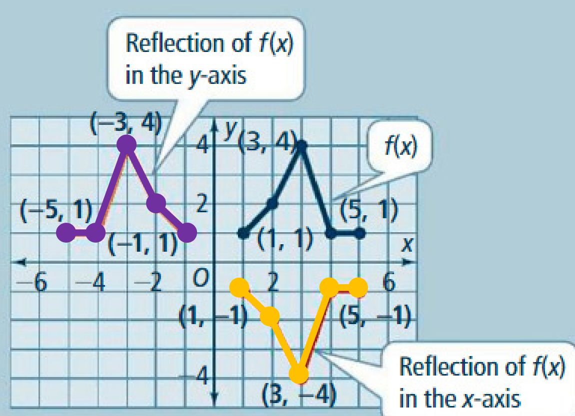
A **reflection** flips the graph of a function across a line, such as the x- or y-axis. Each point on the graph of the reflected function is the same distance from the line of reflection as its corresponding point on the graph of the original function.

When you reflect a graph in the y-axis, the x-values change signs and the y-values stay the same.

When you reflect a graph in the x-axis, the x-values stay the same and the y-values change signs.

For a function $f(x)$, the reflection in the y-axis is $f(-x)$ and the reflection in the x-axis is $-f(x)$.



Problem 3 **Reflecting a Function Algebraically** Page 84

Got It? Let $h(x)$ be the reflection of $f(x) = 3x + 3$ in the x-axis. What is a function rule for $h(x)$?

Page 84

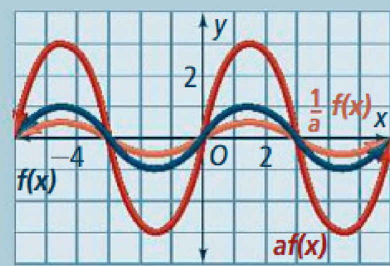
Practice **Write the function rule for each function reflected in the given axis.**

5. $f(x) = 3x$; y-axis

6. $f(x) = 2x - 4$; x-axis

Page 84

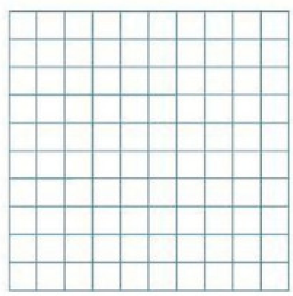
A **vertical stretch** multiplies all y-values of a function by the same factor greater than 1. A **vertical compression** reduces all y-values of a function by the same factor between 0 and 1. For a function $f(x)$ and a constant a, $y = af(x)$ is a vertical stretch when $a > 1$ and a vertical compression when $0 < a < 1$.



Problem 4 **Stretching and Compressing a Function** Page 85

Got It? **a.** The table at the right shows the function $f(x)$. What are the corresponding table and graph for the transformation $h(x) = \frac{1}{3}f(x)$?

x	$f(x)$
−5	2
−2	2
0	−3
3	1
5	−2



b. Reasoning If several transformations are applied to a graph, will changing the order of transformations change the resulting graph? Explain.

Practice **Write an equation for each transformation of $y = x$.**

7. vertical stretch by a factor of 4

8. vertical compression by a factor of $\frac{1}{4}$

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take note

Concept Summary — Transformations of $f(x)$

Vertical Translations

Translation up k units, $k > 0$
$y = f(x) + k$
Translation down k units, $k > 0$
$y = f(x) - k$

Horizontal Translations

Translation right h units, $h > 0$
$y = f(x - h)$
Translation left h units, $h > 0$
$y = f(x + h)$

Vertical Stretches and Compressions

Vertical stretch, $a > 1$
$y = af(x)$
Vertical compression, $0 < a < 1$
$y = af(x)$

Reflections

In the x-axis
$y = -f(x)$
In the y-axis
$y = f(-x)$

Problem 5 Combining Transformations Page 86

Got It? **a.** The graph of $g(x)$ is the graph of $f(x) = x$ stretched vertically by a factor of 2 and then translated down 3 units. What is the function rule for $g(x)$?

b. What transformations change the graph of $f(x) = x^2$ to the graph of $g(x) = (x + 4)^2 - 2$?

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Practice

9. Write the function rule $g(x)$ after a translation up 5 units followed by a reflection in the x-axis of the graph of the function $f(x) = 4x$.

10. Describe the transformations of the function $f(x) = 3x$ that produce the function $g(x) = \left(\frac{3x}{4} - 2\right)$.