AP Calculus AB **1.5 Inverse Functions and Logarithms**

Name:____Per:_________Date:____________________

To complete these notes, go to www.BishSoft.org and watch the video for section 1.5.

Inverse Functions

In technical jargon, an inverse of a function maps the elements of the range to the elements of the domain. In English, this means that the inverse of a function reverses the domain and range. Not all graphs were defined as functions, and we had the **Vertical Line Test** to determine whether a graph was or was not a function. Similarly, not all functions have an inverse that is a function, and we have the **Horizontal Line Test** to determine whether or not a given function has an inverse function.

Definition: One-to-One Function

A function $f(x)$ is **one-to-one** on a domain *D* if $f(a) \neq f(b)$ whenever $a \neq b$.

A function that is **one-to-one** has an inverse.

The definition above can be seen graphically with the use of a horizontal line test. If there are two *x*-values for any given *y*-value of function, then the function does **NOT** have an inverse.

Example 1. Does $y = x^2 + 5x$ have an inverse? Why or why not?

Example 2. Does $y = x^3 + x$ have an inverse? Why or why not?

Once we know whether a function has an inverse, our next task is to find an equation and/or a graph for the inverse.

Finding the Inverse Graphically (two ways)

1. Reflect the graph of the original function over the line $y = x$.
2. Plot the reverse of the coordinates.

Finding the Inverse Algebraically

Switch the *x* and *y* in the original equation, then solve the new equation for *y* in order to write *y* as a function of *x*.

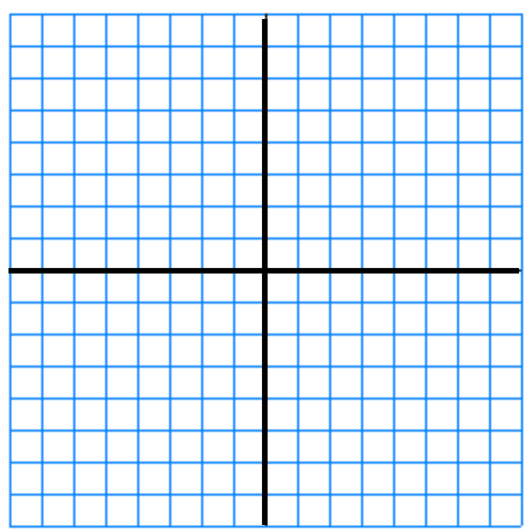
Example 3. Let $f(x) = x^3 - 1$.

A. Graph the function on the grid to the right.

B. Draw the line $y = x$.

C. Reflect the graph of $f(x)$ over the line $y = x$.

D. Find the inverse of the function algebraically.



E. Use your graphing calculator to verify your answer to part ***D***.

Verifying/Proving Inverses

It is one thing to ***find*** the inverse function (either graphically or algebraically), but it is another to ***verify*** that two functions are actually inverses. Whenever you are verifying anything in mathematics, you must go back and use the definition.

Definition: Inverse Function

A function $f(x)$ has an inverse $f^{-1}(x)$ if and only if $f\left(f^{-1}(x)\right) = x = f^{-1}\left(f(x)\right)$.

Example 4. According to this definition, how many composite functions must you use to check whether or not two functions are inverses of each other?

Example 5. Find $f^{-1}(x)$ and verify if $f(x) = \dfrac{x+3}{x-2}$.

Logarithmic Functions

How do Logarithms fit into this discussion? A logarithmic function is just the inverse of an exponential function.

Example 6. Graph $y = 2^x$ and find the inverse of the function graphically.

The equation of the inverse function is:

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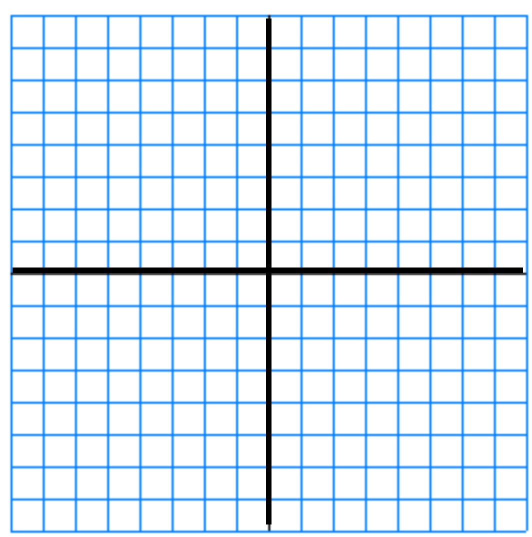

Properties of Logarithms

Definition of a Logarithm
$\log_b(y) = x \Leftrightarrow b^x = y$

Inverse Properties of a logarithm
$b^{\log_b(x)} = x$ and $\log_b b^x = x$

Product Property
$\log_b(xy) = \log_b(x) + \log_b(y)$

Power Property
$\log_b(x^y) = y \bullet \log_b(x)$

Quotient Property
$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

Change of Base Formula
$\log_b(x) = \frac{\ln(x)}{\ln(b)}$

Other Properties
$\log_b(b) = 1$
$\log_b 1 = 0$

Example 7. Evaluate the following without using your calculator.

A. $\log_2\left(\frac{1}{8}\right) =$

B. $\log_{27} 9 =$

Example 8. Solve for x in the following equations.

A. $\log_2(x-1) = 5$

B. $3\left(5^{x-1}\right) = 86$

C. $\log(8x) - \log\left(1+\sqrt{x}\right) = 2$