Name:______________________________________________Per:________Date:___________________

**To complete these notes, go to [www.BishSoft.org](http://www.BishSoft.org) and watch the video for section 1.2.**

## Functions

In the last section we discussed lines and when we needed to write "*y* as a function of *x*". But what is a function?

In Algebra 1, we defined a function as a rule that assigned one and only one (a unique) output for every input. We called the input the **Domain** and the output the **Range**.

**Definition:** A **Function** from a set $D$ to a set $R$ is a rule that assigns a *unique* element in $R$ to each element in $D$. The **Vertical Line** test is the graphical interpretation of this definition.

**Example 1.** There are 3 domain restrictions you MUST continue to be aware of throughout this course. What are they?

## Intervals

In this course we would like not only to know what the domain and range are, but how to describe them with the correct notation. The domain and range of a function could be all real numbers, or we may need to limit the domain and/or the range using **intervals** that are either **open** or **closed**.

Open and closed intervals have endpoints. If the **endpoint is included**, then we say the interval is **closed** at that point, and if the **endpoint is not included,** then we say the interval is **open** at that point. We use a parentheses, $(a, b)$, to indicate open, and a bracket, $[a, b]$, to indicate closed.

**Example 2.** Use interval notation AND inequality notation to describe each interval on the *x*-axes below.


While it is many times possible to just look for the restrictions to the domain, the range of a function is easier to determine if you have a graph or you know what the graph looks like. You should know what the following basic functions look like without having to use your calculator. Can you draw an accurate sketch with at least 3 points on each?

$y = x$ $\quad$ $y = x^2$ $\quad$ $y = x^3$ $\quad$ $y = \sqrt{x}$ $\quad$ $y = \dfrac{1}{x}$

$y = \dfrac{1}{x^2}$ $\quad$ $y = |x|$ $\quad$ $y = \lfloor x \rfloor$ $\quad$ $y = ab^x, 0 < b < 1$ $\quad$ $y = ab^x, b > 1$

$y = \log_b(x), b > 1$ $\quad$ $y = \sin(x)$ $\quad$ $y = \cos(x)$ $\quad$ $y = \tan(x)$

Do you know the domain and range of each of these functions?

## Even and Odd Functions

Recognizing the behavior of functions is not limited to their domain and range. Many functions have the symmetric property of being odd or even. You need to be able to recognize the graph of a function as odd or even, ***and*** you need to understand how to show/verify/prove that a function is even or odd algebraically.

### Graphical Recognition of Even and Odd Functions

An **EVEN** function is ***symmetrical about the y-axis***. Example: $y = \cos(x)$

An **ODD** function is ***symmetrical about the origin***. Example: $y = \sin(x)$

### Algebraic Properties of Even and Odd Functions

An **EVEN** function has the property that $f(-x) = f(x)$.

That is, if you plug in "–*x*" into the function and simplify, you will obtain the ***original*** function.

An **ODD** function has the property that $f(-x) = -f(x)$.

That is, if you plug in "–*x*" into the function and simplify, you will obtain the ***opposite of the original*** function.

**Example 3.** Prove whether the following functions are even, odd, or neither.

**A.** $g(x) = x^3 - x$

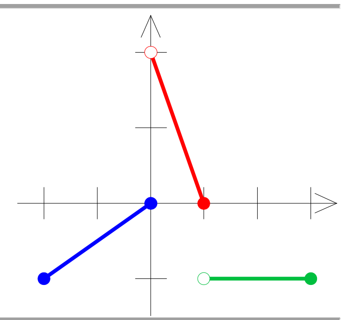
**B.** $h(x) = 1 + \cos(x)$

## Piecewise Functions

Some functions are broken into pieces and behave differently depending on the restricted domain of each piece. Such functions are called piecewise functions. An example of a function that can be written as a piecewise function is the absolute value function $f(x)=|x|$. Be sure to use correct domain restrictions.

**Example 4.** Sketch $f(x)=|x|$, and write an equation for the two "pieces" using a domain appropriate to each piece.

**Example 5.** Write a piecewise function for the graph at the right.



## Composite Functions

When the range of one function is used as the domain of a second function we call the entire function a composite function. Because of the range of the first function is used as the domain of the second, you must not assume the final function has the same domain and range as it would have had if written independently.

We use the notation $(f \circ g)(x)=f(g(x))$ to describe composite functions. This is read "*f* composed with *g*" or "*f* of *g* of *x*".

**Example 6.** If $f(x)=1-x^2$ and $g(x)=\sqrt{x}$, find $g(f(x))$. What is the domain and range of $g(f(x))$?

**Example 7.** If $f(x)=\dfrac{2x-1}{x+3}$, and $g(x)=\dfrac{3x+1}{2-x}$, find $f(g(x))$. Based on your answer, how might *f* and *g* be related?